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Positivity contraints on initial spin observables in inclusive reactions

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Abstract

For any inclusive reaction of the type $A_1(\text{spin}1/2) + A_2(\text{spin}1/2) \to B + X$, we derive new positivity constraints on spin observables and study their implications for theoretical models in view, in particular, of accounting for future data from the polarized pp collider at BNL-RHIC. We find that the single transverse spin asymmetry A_N for several processes, for example jet production, direct photon production, lepton-pair production, must be such that $|A_N| \lesssim 1/2$, rather than the usual bound $|A_N| \leq 1$.

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Let us consider an inclusive reaction of the type

$$A_1(\mathrm{spin}1/2) + A_2(\mathrm{spin}1/2) \to B + X \ , \tag{1}$$

where the spins of both initial spin1/2 particles can be in any possible directions and no polarization is observed in the final state. The observables of this reaction can be expressed in terms of the discontinuities (with respect to the invariant mass squared of X) of the amplitudes for the forward three-body scattering

$$A_1 + A_2 + \overline{B} \to A_1 + A_2 + \overline{B} \ , \tag{2}$$

as given by the generalized optical theorem. We assume parity conservation, so the complete knowledge of this reaction requires the determination of *eight* real functions, which is the number of independent spin observables [1]. In order to define these observables, we recall the standard notation used in Ref. [2] $(A_1A_2|BX)$, by which the spin directions of A_1, A_2, B and X are specified in one of the three possible directions L, N, S. Since the final spins are not observed, we have in fact $(A_1A_2|00)$ and $\overrightarrow{L}, \overrightarrow{N}, \overrightarrow{S}$ are unit vectors, in the center-of-mass system, along the incident momentum, along the normal to the scattering plane, and along $\overrightarrow{N} \times \overrightarrow{L}$, respectively. In addition to the unpolarized cross section $\sigma_0 = (00|00)$, there are *seven* spin dependent observables, *two* single transverse spin asymmetries

$$A_{1N} = (N0|00) \quad \text{and} \quad A_{2N} = (0N|00) \ , \tag{3}$$

and *five* double-spin asymmetries

$$\begin{aligned} A_{LL} &= (LL|00) \ , \ A_{SS} = (SS|00) \ , \ A_{NN} = (NN|00) \ , \\ &A_{LS} = (LS|00) \ \text{ and } \ A_{SL} = (SL|00) \ . \end{aligned} \tag{4}$$

If $\overrightarrow{e}_1$ and $\overrightarrow{e}_2$ are the polarization unit vectors of A_1 and A_2, the corresponding polarized cross section is

$$\sigma(\overrightarrow{e}_1, \overrightarrow{e}_2) = \mathrm{Tr}(M\rho) \ . \tag{5}$$

In this expression M denotes the spin scattering matrix and ρ is the 4×4 density matrix $\rho = 1/4(\mathbf{1} + \overrightarrow{e}_\mathbf{1} \cdot \overrightarrow{\sigma}_\mathbf{1}) \cdot (\mathbf{1} + \overrightarrow{e}_\mathbf{2} \cdot \overrightarrow{\sigma}_\mathbf{2})$, where $\overrightarrow{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ stands for the three 2×2 Pauli matrices and $\mathbf{1}$ is the 2×2 unit matrix; M is also a 4×4 matrix, which we shall parametrize in the following way †

$$\begin{aligned} M &= \sigma_0[\mathbf{1} \cdot \mathbf{1} + A_{1N}\sigma_{1z} \cdot \mathbf{1} + A_{2N}\mathbf{1} \cdot \sigma_{2z} + A_{NN}\sigma_{1z} \cdot \sigma_{2z} + \\ & \quad A_{LL}\sigma_{1x} \cdot \sigma_{2x} + A_{SS}\sigma_{1y} \cdot \sigma_{2y} + A_{LS}\sigma_{1x} \cdot \sigma_{2y} + A_{SL}\sigma_{1y} \cdot \sigma_{2x}]. \end{aligned} \tag{6}$$

This expression is fully justified, since we have explicitly

$$\begin{aligned} \sigma(\vec{e}_1, \vec{e}_2) &= \sigma_0[1 + A_{1N}e_{1z} + A_{2N}e_{2z} + A_{NN}e_{1z}e_{2z} + \\ & \quad A_{LL}e_{1x}e_{2x} + A_{SS}e_{1y}e_{2y} + A_{LS}e_{1x}e_{2y} + A_{SL}e_{1y}e_{2x}] \, . \end{aligned} \tag{7}$$

The crucial point is that M is a Hermitian and *positive* matrix, leading to the following *two* strongest constraints ‡

$$(1 \pm A_{NN})^2 \geq (A_{1N} \pm A_{2N})^2 + (A_{LL} \pm A_{SS})^2 + (A_{LS} \pm A_{SL})^2 \; , \tag{8}$$

which are necessary and sufficiant. As special cases of Eq. (8), we have the *six* weaker constraints

$$1 \pm A_{NN} \geq |A_{1N} \pm A_{2N}| \; , \tag{9}$$

$$1 \pm A_{NN} \geq |A_{LL} \pm A_{SS}| \; , \tag{10}$$

and

$$1 \pm A_{NN} \geq |A_{LS} \pm A_{SL}| \; . \tag{11}$$

These constraints are very general and must hold in any kinematical region and for many different situations such as electron–proton scattering, electron–positron scattering or quark–quark scattering, but we now turn to a specific case, which is

† A much simpler form was used in the case of the pp total cross section, in pure spin states, to derive positivity bounds [3].

‡ Similar constraints were obtained in Ref. [4] for depolarization parameters corresponding to the spin transfer between one initial spin1/2 particle and one final spin1/2 particle. For constraints on spin observables in nucleon–nucleon elastic scattering and in the strangeness-exchange reaction $\bar{p}p \to \bar{\Lambda}\Lambda$, see Refs. [5, 6].

of direct relevance to the spin programme at the BNL-RHIC polarized pp collider [7]. In a proton–proton collision, since the initial particles are identical, we have $A_{1N} = A_{2N} \equiv A_N$ and $A_{LS} = A_{SL}$. In this case, Eq. (8) with the + sign reduces to

$$(1 + A_{NN})^2 \geq 4A_N^2 + (A_{LL} + A_{SS})^2 + 4A_{SL}^2 \ . \tag{12}$$

This implies in particular

$$1 + A_{NN} \geq 2|A_N| \ , \tag{13}$$

and

$$1 + A_{NN} \geq 2|A_{SL}| \ , \tag{14}$$

so that, the allowed range of A_N and A_{SL} is strongly reduced, if A_{NN} turns out to be large and negative. Conversely if $A_{NN} \simeq 1$, these constraints are useless. Note that, in the kinematical region accessible to the pp polarized collider, a calculation of A_{NN} for direct photon production and jet production has been performed [8]; it was found that $|A_{NN}|$ is of the order of 1 or 2%. Similarly, based on Ref. [9], this double transverse spin asymmetry for lepton pair production was estimated to be a few per-cent [10]. The direct consequence of these estimates is that $|A_N|$ and $|A_{SL}|$, for these processes[§], are essentially bounded by 1/2. In addition, from Eq. (10), there are two other non-trivial constaints: $1 \geq |A_{LL} \pm A_{SS}|$.

Single transverse spin asymmetries in inclusive reactions at high energies are now considered to be directly related to the transverse momentum of the fundamental partons involved in the process. This new viewpoint, which has been advocated to explain the existing data in semi-inclusive deep inelastic scattering [12, 13], will have to be more firmly established also by means of future data from BNL-RHIC. On the theoretical side several possible leading-twist QCD mechanisms [14, 15] have been proposed to generate these asymmetries in leptoproduction [16, 17], but also in pp collisions. We believe that these new positivity constraints on spin observables for

[§]It is amusing to recall that, using a phenomenological approach for lepton-pair production, bounds on $|A_N|$ larger than 50% were obtained in Ref. [11], but at that time it was not known that A_{NN} is small.

a wide class of reactions will be of interest for model builders as well as for future measurements.

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References

[1] G. R. Goldstein and J. F. Owens, Nucl. Phys. **B103**, 145 (1976).

[2] C. Bourrely, E. Leader and J. Soffer, Phys. Rep. **59**, 95 (1980).

[3] J. Soffer and D. Wray, Phys. Lett. B **43**, 514 (1973).

[4] M. G. Doncel and A. Méndez, Phys. Lett. B **41**, 83 (1972).

[5] C. Bourrely and J. Soffer, Phys. Rev. D **12**, 2932 (1975).

[6] M. Elchikh and J. M. Richard, Phys. Rev. C **61**, 035205 (2000).

[7] G. Bunce, N. Saito, J. Soffer and W. Vogelsang, Annu. Rev. Nucl. Part. Sci. **50**, 525 (2000).

[8] J. Soffer, M. Stratmann and W. Vogelsang, Phys. Rev. D **65**, 114024 (2002); A. Mukherjee, M. Stratmann and W. Vogelsang, hep-ph/0303226.

[9] J. Soffer, Phys. Rev. Lett. **74**, 1292 (1995).

[10] O. Martin, A. Schäfer, M. Stratmann and W. Vogelsang, Phys. Rev. D **60**, 117502 (1999).

[11] J. Soffer and P. Taxil, Nucl. Phys. **B172**, 106 (1980).

[12] SMC Collaboration, presented by S. Bravar, Nucl. Phys. B (Proc. Suppl.) **79**, 520 (1999).

[13] HERMES Collaboration, A. Airapetian *et al.*, Phys. Rev. Lett. **84**, 4047 (2000); Phys. Rev. D**64**, 097101 (2001).

[14] D. Sivers, Phys. Rev. D **41**, 83 (1990) and D **43**, 261 (1991).

[15] J. C. Collins, Nucl. Phys. **B396**, 161 (1993).

[16] S. J. Brodsky, D. S. Hwang and I. Schmidt, Nucl. Phys. **B642**, 344 (2002).

[17] M. Anselmino, U. D'Alesio and F. Murgia Phys. Rev. D **67**, 074010 (2003).